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Lateral diffusion of large particles in turbulent pipe flow

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A closed flow circuit, comprising a smooth horizontal plastic pipe transporting a dilute suspension of plastic particles in water, was modified to produce a satisfactory environment for the detection of mean particle location across a fixed plane normal to the mean turbulent flow direction. It was established that mean concentration in the vicinity of the vertical diameter was a function of the vertical co-ordinate only, and this result enabled the particle diffusivity to be evaluated for a core region of the cross-section. For a particle relative density in the range 1·035 to 1·126 and a variety of flows, a suitable expression for the dimensionless particle diffusivity is $A(1-BW/u_*)S^N$, where u_* is the friction velocity, W the fall velocity for a particle in still water, S the relative density of the suspended material, and A, B and N are constants.

1. Introduction

Previous workers (Batchelor, Binnie & Phillips 1955; Binnie & Phillips 1958; Barnard & Binnie 1963; Batchelor 1965) have stimulated the writers' interest in the problem of evaluating the lateral diffusivity of large discrete particles in turbulent suspension.

The Cambridge studies of Binnie, Phillips and Barnard involved timing heavy spherical particles of 0·2 in. diameter over a measured length of a 2 in. diameter pipe, in which the mean flow velocity was approximately 5 ft/sec. The mean particle velocity determined from a large number of realizations was found for a range of densities. The mean particle velocity was also predicted on the basis of the assumption, first, that equality of local particle and fluid longitudinal velocities holds in any given small region of the cross-section and, secondly, that the particle concentration is a function of vertical position only. An unknown 'vertical diffusivity coefficient' applying over the accessible cross-section was estimated by comparing the experimental and calculated mean particle velocities. Barnard & Binnie, in addition, conducted experiments to determine 'mean vertical diffusivity' from photographic observations of particle locations across an illuminated cross-section.

Recent developments using a suspension wire detection device (Sharp & O'Neill 1968) have shown a considerable advance over optical methods in obtaining the necessary number of discrete observations at a given cross-section. As the time of total observation is thus effectively increased a commensurate improvement in the flow circuit controls was found necessary.

The present experiments used 0·1 in. diameter particles in a 2 in. diameter plastic pipe, and ranges of flow conditions and particle densities were examined.

One aspect given considerable attention in the studies was the exact nature of the environment associated with the particle transport. Assumptions such as two-dimensionality have been made by others. It was found necessary for the present purposes to designate a number of distinct zones in the pipe cross-section.

2. Experimental facility

The facility consisted of a closed circuit as illustrated in figure 1 with a 2 in. diameter test section into which particles were fed at a random rate. Less than 0·1 % volumetric concentration was achieved, and each particle could be considered as acting alone.

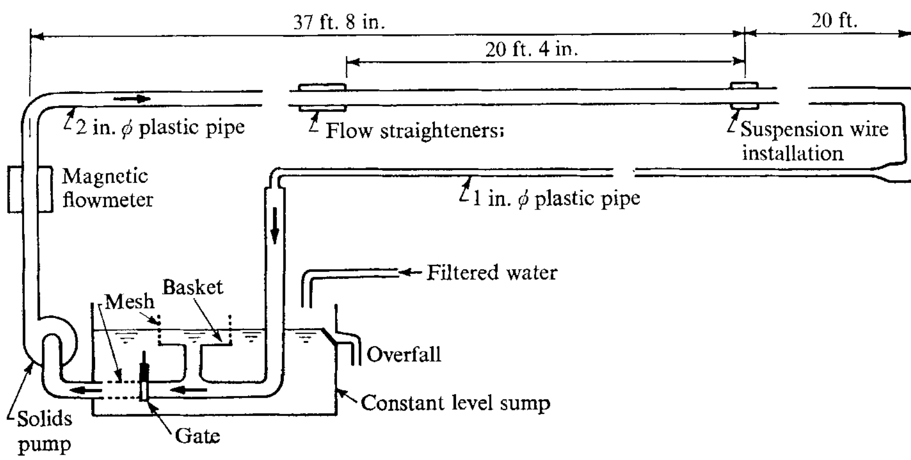

FIGURE 1. Schematic diagram of flow circuit.

The flow and temperature were maintained constant over long periods, enabling long-term averages to be determined.

The detection technique incorporates the suspension wire flow measuring device (Sharp 1964) as the sensor for registering particle impact along a measured exposed length of fine wire. The ringing signal (from an individual impact) is rectified, filtered and recorded or counted as a single event and the accumulation of many such events in a prescribed time measures the impacts per unit time for a given length of wire. The exposed length of wire is varied so that the measurement may be repeated for a finite number of positions over the full diameter. The suspension wire device was rotated and measurement also made along diameters other than the vertical.

The facility was able to detect impact signals in magnitude as low as the turbulence registered by the suspension wire but the steeper leading edge of the impact signal enabled the counter to discriminate the occasional small impact. In the absence of particles in the circuit it was demonstrated that biasing the output voltage of the filter to partly upset the diode function enabled the counter to register a number of counts associated with turbulent fluctuations. However, these conditions were readily avoided during impact tests.

The plastic particles were 0·1 in. nominal diameter injection moulded in a die capable of producing 150 simultaneously and had the characteristics detailed in the table below. The fall velocities of the particles were obtained from timings recorded over a 10 ft. measuring section in a 9 in. diameter vertical cylinder. The particles were checked after long periods of immersion in water and continued use in the flow circuit and showed no significant change in characteristics.

Trade name	Relative density	Mean fall velocity at 21 °C W in./sec	Statistical parameters		
			C_v	α_3	$\alpha_4 - 3$
ABS	1·035	1·122	0·135	−0·0862	0·0873
SAN	1·069	1·838	0·055	0·1261	0·2014
Nylon	1·126	2·843	0·014	—	—

The statistical parameters C_v, α_3, and $\alpha_4 - 3$ refer to the coefficients of variation, skewness and kurtosis respectively.

3. Preliminary investigations

Preliminary tests were conducted to establish the most suitable set of conditions for operation of the detection technique and to enable the mean relative concentration distribution to be evaluated within a range of experimental variation of flow and particle type.

Friction loss measurements indicated that 'smooth pipe' conditions existed and a bar pitot traverse at the test section for several orientations indicated symmetry of the mean velocity profile and, in the core region at least, agreed with the expected velocity profile.

The test section was located 226 pipe diameters downstream of a 90° bend and ordinarily would be considered free of undesirable circulations in the flow and the bar pitot showed no evidence of such circulations. However, a preliminary traverse over the whole cross-section with the lightest particles (relative density, $S = 1{\cdot}035$) revealed the pattern of relative concentration shown in figure 2, suggesting that the approach length for a fully developed undisturbed mean velocity profile was insufficient for a study which required an environment free of secondary flows. The decay of secondary flows due to bends is not precisely known (Kreith & Sonju 1965) but an estimate of a residual of 1 % of the mean longitudinal velocity in 200 pipe diameters gives a value of the same order of magnitude as the particle fall velocity, and suggests that the result in figure 2 is not unreasonable. That is, the total distribution of particles is a very sensitive indication of the presence of secondary flow which would not be detected by normal methods.

In order to determine the diffusive property of the medium these weak secondary flow effects had to be eliminated and the installation of straightening vanes 122 pipe diameters upstream of the test section accomplished this as figure 3 indicates for subsequent tests of the lower section.

In addition, traverses at 15° to the vertical in the upper region of the pipe cross-section confirmed that the concentration in a relatively broad strip encompassing the vertical diameter was a function of vertical co-ordinate only.

4. Experimental programme

Turbulence measurements (Laufer 1954) and point source diffusion studies (Baldwin & Mickelsen 1963; Flint, Kada & Hanratty 1960) indicate that the core region of a pipe flow can be considered with some reservations to be approximately a homogeneous, isotropic, turbulent flow. The above work together with the observed concentration distribution (figure 3) suggests the flow may be given the physical description in figure 4.

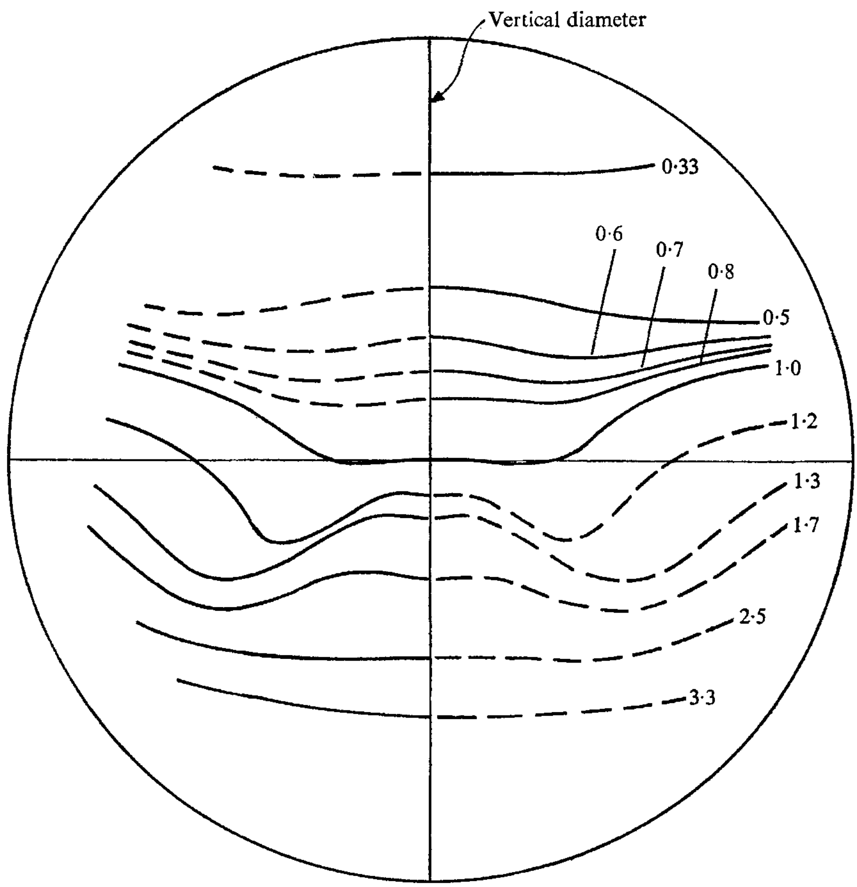

FIGURE 2. Relative concentration distribution, C/C_0, indicating influence of secondary flow. The distribution is based on 200 test runs. $U = 3{\cdot}5$ ft/sec. 0·1 in. ϕ ABS spheres.

In figure 4 various zones have been given approximate dimensions. Zone A is regarded as the core region where the time-averaged concentration is a function of diffusion and gravity alone. Zone B is a region where shear forces become significant. Zone C is a region of limited accessibility for the measurement procedures used.

Most work in the experimental programme was devoted to the core zone where it was anticipated that the simplest flow environment existed. The majority of traverses were along a vertical diameter although some traverses were also taken

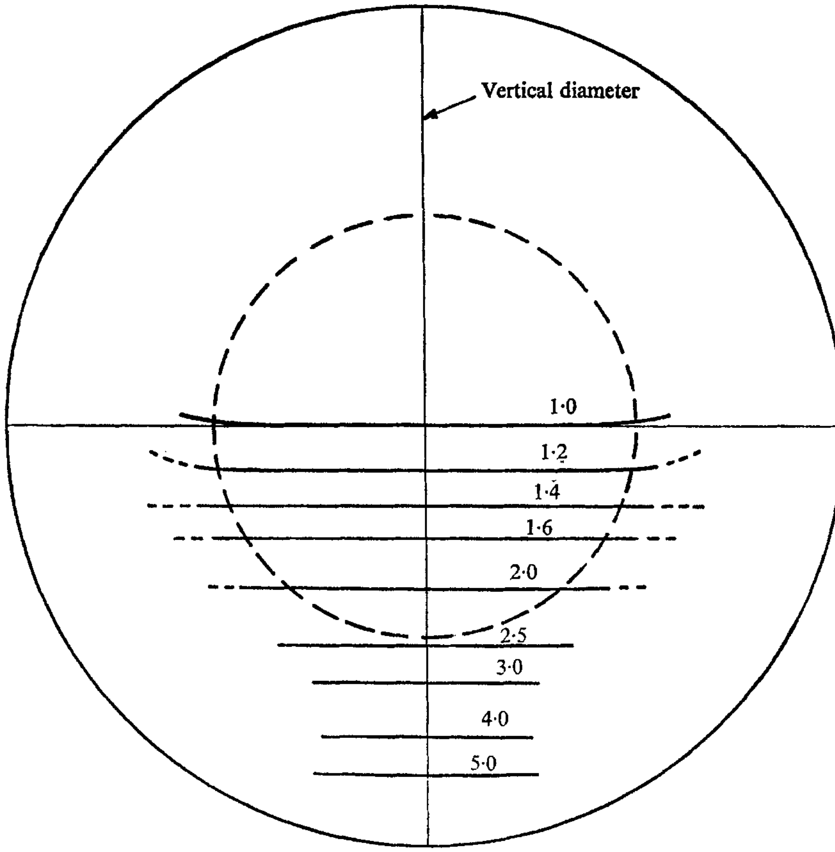

FIGURE 3. Relative concentration distribution, C/C_0, flow straighteners installed in circuit. The distribution is based on 120 test runs. $U = 3{\cdot}5$ ft/sec. 0·1 in. ϕ ABS spheres.

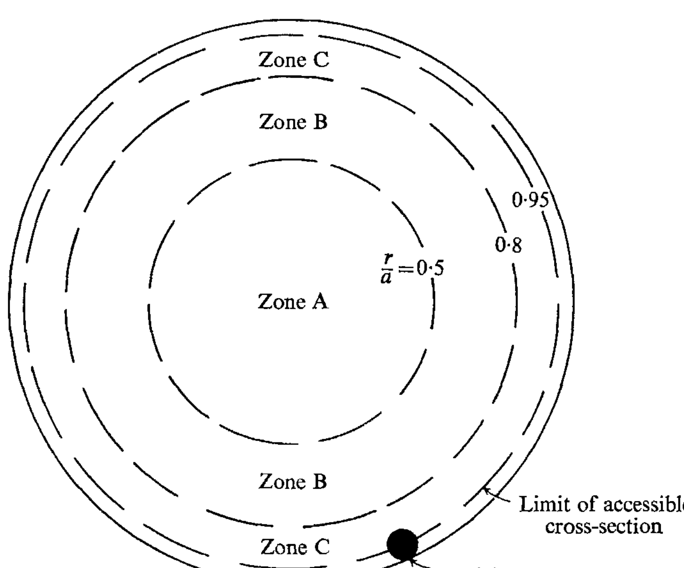

FIGURE 4. Classification of zones in the cross-section.

along a horizontal diameter. To obtain a sufficiently accurate time-averaged rate of impacts per second for a given wire exposure length it was generally necessary to use recording times of between 10 and 30 min. Flow control was extremely important and both stable temperature and accurate pump control (0·1 %) were achieved for a day's test.

To make a complete traverse along a diameter required more than a working day and in general traverses were therefore limited to portion of a diameter. Conditions were constant during each test and thus relative concentrations could be determined. For each of the three particle types a range of mean flow velocity within the range 2–5½ ft/sec was used.

The investigation was confined to an elucidation of the respective roles of diffusion, gravity, inertia and pressure forces in the environment described above.

5. Mass flow rate distribution and diffusivity

The detection device yields a cumulative time-averaged mass flow rate which represents the rate at which particle centres are passing through a portion of the flow cross-section closely determined by the length of exposed wire and the particle diameter. Differentiation provides a measure of the relative mean particle mass flow rate variation with position.

The relative concentration distribution was derived by dividing the relative mass flow rate of particles by the local liquid mass flow rate assuming a suitable velocity distribution (Taylor 1954) and also that particle slip is not significant in this context. The relative concentration was assigned a value of unity at the pipe centre.

The preliminary studies indicating C as a function of vertical co-ordinate (y) only, invites the use of the well-known differential equation which describes a balance between gravity fall-out and an upward flux caused by the mixing processes between horizontal planes:

$$\epsilon_p dC/dy + WC = 0. \tag{1}$$

This traditional form has usually been applied with W as the fall velocity of a particle in still water and this will be discussed later. The solution of the equation over any region where the coefficient of particle diffusion ϵ_p is constant gives

$$\frac{C}{C_a} = \exp\left[-\frac{W}{\epsilon_p}(y-y_a)\right], \tag{2}$$

where a refers to an arbitrary reference position in the region.

The gradient diffusion process thus described was investigated in the zones A and B. Relative concentration *versus* vertical co-ordinate (y) produced well-defined straight lines on a log–linear representation for the core region for every case. This establishes the suitability of (2) and enables ϵ_p to be evaluated.

It was also found that straight lines of a different slope were appropriate to zone B, suggesting that a similar relationship (2) would be valid for this region, but clearly the presence of a shear field requires additional factors to be considered in order to specify the nature of the diffusion process throughout the medium. An examination of zone B is described elsewhere by the authors (Sharp & O'Neill 1970).

In zone A the value of ϵ_p was calculated using the 'representative' fall velocity corresponding to the mean fall velocity W in still water.

6. Evaluation of diffusivity in core region A

The approximation of the core region to a homogeneous (and perhaps isotropic) flow field, as indicated by turbulence measurements and heat and fluid mass diffusion studies by others, supports the use of a gradient-type diffusion process involving a constant scalar coefficient for given flow conditions. The quantitative support for the gradient type diffusion assumption (and hence the application of ϵ_p as a diffusion coefficient) is provided by these experiments.

The values of ϵ_p are shown in figure 5 for the three particle types and a range of values of u_*. A linear variation between ϵ_p and u_* is observed which, however, differs from the Reynolds analogy in that ϵ_p does not vanish with u_* and the values of ϵ_p are significantly greater. It is clear that ϵ_p is a function of particle size and the density difference between the particle and the ambient fluid.

The difference between the particle and fluid densities causes two effects to arise: those due to gravity and those due to inertia. If it is assumed that the influence of gravity forces for a given-sized particle is characterized by the fall velocity W, then the variation of ϵ_p with density for a given u_* as indicated in figure 5 will be caused by the differences between the particle inertia and that of a corresponding lump of fluid.

The influence of particle inertia is extremely complex although physical reasoning would indicate that increased particle density and hence inertia would lead to a reduction in the diffusion coefficient. Such a trend is apparent in the present experimental results.

Others (Binnie and co-workers) have introduced the dimensionless particle diffusivity $\zeta = 2\epsilon_p/u_*D$, where u_* is the friction velocity and D is the pipe diameter. Figure 6 is a plot of the data of figure 5 in this form. It was found possible to generalize the results in the form

$$\zeta = A(1-BW/u_*)\,S^N, \qquad (3)$$

where W is the fall velocity for a particle in still water, S the relative density of the suspended material, and A, B and N are constants.

For the 0·1 in. diameter particles used in the present studies $A = 0{\cdot}38$, $B = 0{\cdot}52$ and $N = 2{\cdot}8$. In this equation ζ is not greatly altered by the choice of N. The range of experiments leading to this result is shown in figure 7, where the constant A in (3) is evaluated.

If it is assumed that the values of A, B, and N remain constant as S approaches unity, the value of ζ for a neutrally buoyant particle is then deduced as

$$\zeta = 0{\cdot}38. \qquad (4)$$

The value of 0·38 is considerably greater than the core value (say 0·08) required by the Reynolds analogy. A similar trend may be noticed in the experiments of

Barnard & Binnie (1963) to determine 'vertical diffusivity' for 0·2 in. diameter spheres for given flow conditions in a 2 in. diameter pipe.

It is known that small particles in flume experiments indicate local particle diffusion coefficients of magnitude approximately equal to the local eddy viscosity (Vanoni 1946; Jobson & Sayre 1970). The authors are unaware of any satisfactory explanation of the discrepancy. It would appear that particle size plays an important role apart from simply specifying (along with density

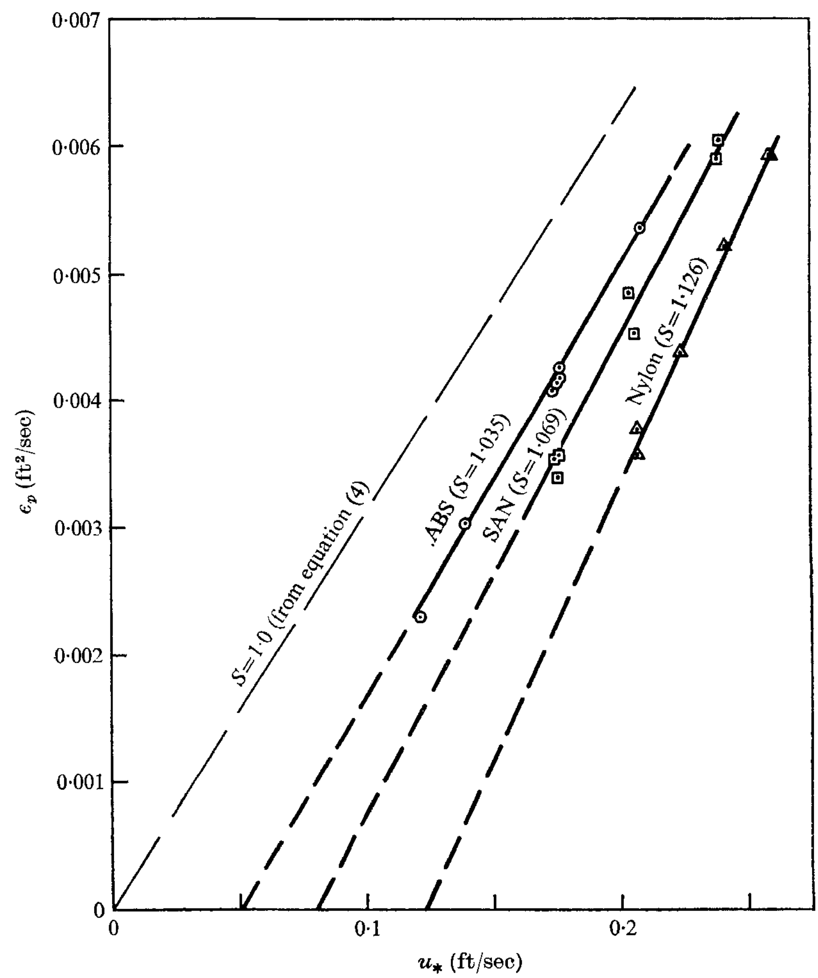

FIGURE 5. Coefficient of diffusion (core region) for 0·1 in. diameter spheres *vs*. friction velocity.

difference) the spherical particle characteristics which would be important in relation to gravity and inertia effects on the particle behaviour in a given flow field. Batchelor (1965) indicated that it is probable that W (the fall velocity of a particle in still water) may be only roughly representative of the drift due to gravity and that a coefficient α should be introduced in (1) to read

$$\epsilon_p dC/dy + \alpha WC = 0. \qquad (5)$$

Studies of the fall velocity of a particle in a vertically oscillating fluid (Houghton 1966) suggest a possible mechanism contributing to a value of α less than unity. However, for the present experiments it appears unlikely that this effect, resulting from non-linear particle–fluid interaction, could alone lead to values

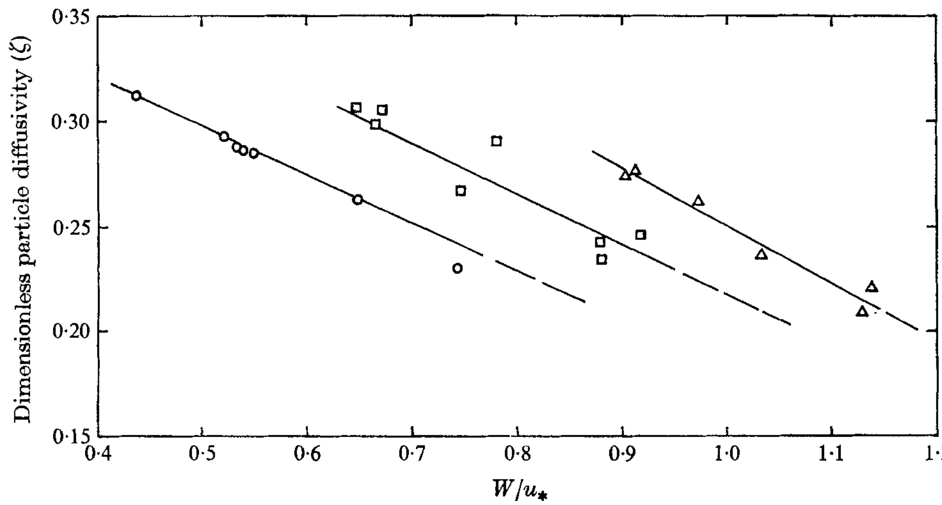

FIGURE 6. Diffusivity for 0·1 in. diameter spheres *vs.* W/u_*.
○, ABS; □, SAN; △, Nylon.

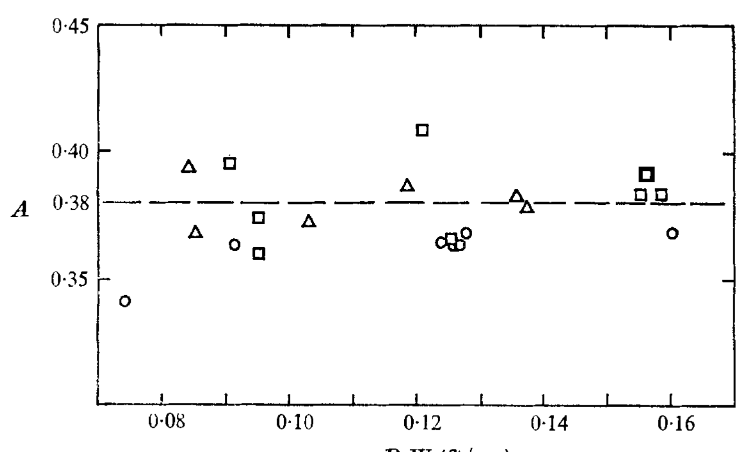

FIGURE 7. Scatter of test results in evaluating A in equation (3).
○, ABS; □, SAN; △, Nylon.

of α sufficiently low to account for the high diffusivities obtained. Apparently conflicting evidence concerning the influence of turbulence on particle drift (Davidson, Pearson & Vanoni 1969) indicates that further studies are needed before the effect of this phenomenon on the value of α can be assessed.

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